REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)* January 2015	**2. REPORT TYPE** Technical Paper	**3. DATES COVERED** *(From - To)* January 2015-May 2015

4. TITLE AND SUBTITLE COMPUTATIONAL MODELING APPROACHES FOR STUDYING TRANSVERSE COMBUSTION INSTABILITY IN A MULTI-ELEMENT INJECTOR	**5a. CONTRACT NUMBER** In-House
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S) Joshua D. Hodson and H. Heath Dewey	**5d. PROJECT NUMBER**
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER Q0A1
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Air Force Research Laboratory (AFMC) AFRL/RQRC 10 E. Saturn Blvd. Edwards AFB, CA 93524-7680	**8. PERFORMING ORGANIZATION REPORT NO.**
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) Air Force Research Laboratory (AFMC) AFRL/RQR 5 Pollux Drive Edwards AFB CA 93524-7048	**10. SPONSOR/MONITOR'S ACRONYM(S)**
	11. SPONSOR/MONITOR'S REPORT NUMBER(S) **AFRL-RQ-ED-TP-2015-154**

12. DISTRIBUTION / AVAILABILITY STATEMENT
Distribution A: Approved for Public Release; Distribution Unlimited.

13. SUPPLEMENTARY NOTES
Technical paper presented at JANNAF, Nashville, TN, 06 June 2015. PA#15282

14. ABSTRACT
The current study describes two modeling approaches to model an unstable seven element linear array of shear coaxial injectors. The first approach is a reduced model where the driving injectors are replaced with an artificial forcing term. The forcing amplitude can be adjusted so that the effect of the transverse instability on the center study element can be examined parametrically. The second approach models the entire domain, and can capture additional details such as the inter-element interactions and the self-excited nature of the instability. Both sets of results are compared with experimental measurements and used to provide physical insights into the underlying instability mechanisms.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT **Unclassified**	**b. ABSTRACT** **Unclassified**	**c. THIS PAGE** **Unclassified**	SAR	15	Doug Talley **19b. TELEPHONE NO** *(include area code)* 661-525-6174

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. 239.18

COMPUTATIONAL MODELING APPROACHES FOR STUDYING TRANSVERSE COMBUSTION INSTABILITY IN A MULTI-ELEMENT INJECTOR

M.E. Harvazinski[1], K.J. Shipley[2*], D.G. Talley[1], V. Sankaran[1], and W.E. Anderson[2]

[1]Air Force Research Laboratory
Edwards AFB, CA

[2] Purdue University
West Lafayette, IN

ABSTRACT

The current study describes two modeling approaches to model an unstable seven element linear array of shear coaxial injectors. The first approach is a reduced model where the driving injectors are replaced with an artificial forcing term. The forcing amplitude can be adjusted so that the effect of the transverse instability on the center study element can be examined parametrically. The second approach models the entire domain, and can capture additional details such as the inter-element interactions and the self-excited nature of the instability. Both sets of results are compared with experimental measurements and used to provide physical insights into the underlying instability mechanisms.

INTRODUCTION

Transverse combustion instabilities have plagued rocket engine design since the 1950's. Lacking a truly predictive model, combustion instability remains a large risk in the design and acquisition of new hardware. The coupling of heat release and flow dynamics may give rise to large pressure fluctuations. This is particularly problematic in confined flows like those found in liquid rocket engines. The coupling between pressure and heat release, known as combustion instability, has been a major problem in the design of rocket engines since the 1950's [1]. Despite many decades of work no model exists which can accurately determine the stability of an engine given the configuration and operating conditions. Early modeling work focused on the use of linear and non-linear wave equation models augmented with a source term to represent the combustion heat release. While this approach has the potential to capture the coupling between the pressure and combustion there is a significant limitation. The model used to describe the heat release must be derived from empirical data; thus, there is no guarantee that it will accurately represent the physics and be truly predictive of combustion instabilities.

The continued expansion and wide availability of high-performance computational resources is allowing computational fluid dynamics (CFD) of unsteady reacting flow simulations to be applied to combustion instability investigations. The highly unsteady nature of the combustion process and coupling with the chamber acoustics dictate need for large eddy or detached eddy simulations as opposed to steady-state Reynolds-Averaged simulations. The simulations are able to reasonable capture the physics involved including acoustics, mixing, combustion, geometrical features and their interactions with each other. The advantage of this approach over the wave equation based analysis is the elimination of the empirical model used capture the relationship between the acoustics and combustion. The computational cost of these simulations remains high, often requiring 1000's of CPU cores used continuously for months.

The computational models can potentially provide a wealth of data about the underlying physics. The ability to compare detailed flowfields for stable and unstable conditions can help to elucidate the physical mechanisms that are responsible for combustion instability. While reacting flow computations are not new their application to combustion instability remains an emerging technology. As such it is important to validate computational results against experimental work. Combined experimental and computational studies of subscale elements is a promising approach

* Presently at the Aerospace Corp.

to both validate simulation tools against available experimental data and to interrogate simulation results to better understand the physical phenomena responsible for combustion instability.

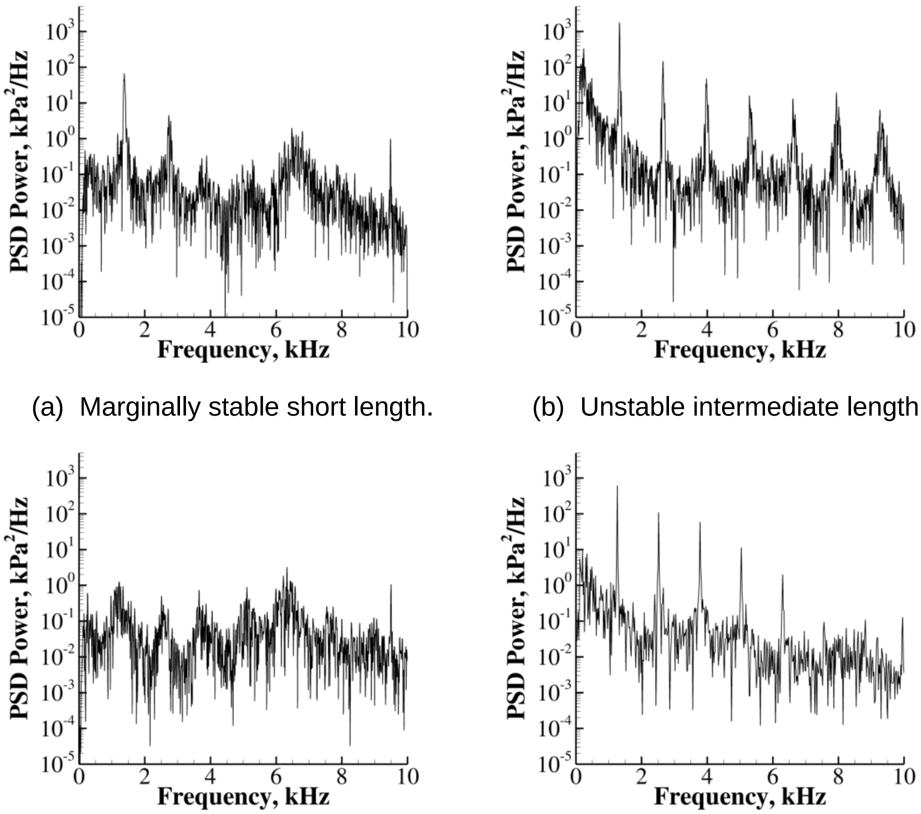
The majority of the published literature on computational modeling of combustion instability has focused on single element configurations [2, 3, 4, 5, 6, 7, 8]. This is partly because of the immense cost associated with even a single element study. While single element studies can provide a wealth of knowledge there are several details which limit their applicability to affect engine design. In a single element configuration there are no inter-element interactions. Production engines will have 100’s or even 1000’s of injectors and it is inappropriate to assume that they will function independent of each other. The effect of the walls is also exaggerated in a single element setup since, in reality, many injectors will not be close to a wall. Moreover, actual rocket engine stabilities often involve transverse modes which can be difficult to study in a single-element configuration.

The focus of the present article is on modeling transverse instability modes in a multi-element configuration. The computations are patterned after companion experiments being investigated at Purdue University. The experiments involve a rectangular chamber that is capable of generating self-excited transverse mode instabilities by utilizing a linear array of injector elements. The injector elements themselves are designed based on the findings of the longitudinal chamber studies and the amplitude of the instabilities is controlled by selectively flowing fuel only in some or all of the injectors. As indicated above, the transverse studies allow for inter-element interactions and realistic characterization of wall effects.

The paper begins with a review of the significant findings of the single element longitudinal mode study. In particular, we discuss the underlying physical phenomena that are responsible for the generation of combustion instabilities in this configuration. The single element injector design is then introduced into a multi-element configuration where transverse instabilities are generated. Two unique approaches to modeling of transverse instability are presented. In the first, a reduced model is simulated where artificial wall-forcing is used to generate the instability. This approach allows only a few of the elements to be modeled. The level of instability is controlled by varying the amplitude of the wall oscillations to match the experimental data. In that manner, the study can provide a detailed assessment of the combustion response of one or more elements to the transverse mode excitation. In the second approach the complete seven element setup is modeled. This enables the self-excited instabilities to be generated and inter-element interactions can be fully captured. In combination, the two methods provide a systematic way to represent transverse mode instabilities and to analyze the underlying physical phenomena.

SINGLE ELEMENT STUDIES

Early single element work theorized that the vortex transport off the dump plane was an important aspect of the instability mechanism. Both simulations and experiments showed that different injector area to the combustor area ratios produced different levels of instability [9, 7]. The continuously variable resonance combustor (CVRC) is a single element model rocket combustor capable of producing varying levels of both stable and unstable combustion. The shear-coaxial injector is subjected to a self-excited longitudinal instability whose amplitude is a function of the oxidizer post length [10]. Combustion is marginally stable at the short lengths and unstable at the intermediate lengths which peak-to-peak pressure variations greater than 40% of the mean. The longest oxidizer post length has experimentally shown both unstable and stable combustion. PSD (power spectral density) plots at each of the four conditions are shown in Figure 1. There is strong harmonic content for the unstable lengths (b) and (d), while only the first two modes are well defined in the marginally stable case (a). The stable case (c) does not have well defined peaks. In each of these cases only the oxidizer post length was varied, while the other operating conditions remained fixed.



(a) Marginally stable short length. (b) Unstable intermediate length.

(c) Stable long length. (d) Unstable long length.

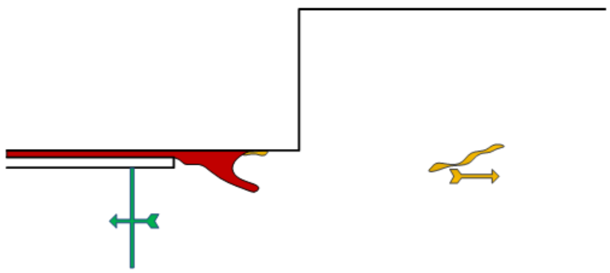
Figure 1: Single element PSD plots showing stable and unstable combustion.

There has been a significant effort in modeling applied to the CVRC experiment. Simulations have been performed in both two axisymmetric setups and full three-dimensional geometries using hybrid RANS-LES and LES. All computational work has been unsteady. The axisymmetric simulations have been largely unable to produce the amplitudes seen in the experiment [6, 2, 3]. Three-dimensional simulations, which are significantly more expensive, have been able to predict amplitudes commensurate with the available experimental data [2, 3, 8, 4]. A notable exception to this was work by Sardeshmukh et al. which used detailed chemical kinetics in an axisymmetric setup and produced amplitudes comparable to the experimental results [5]. All the other simulations have used a global reaction mechanism because a three-dimensional simulation with detailed kinetics remains unaffordable even on current supercomputers.

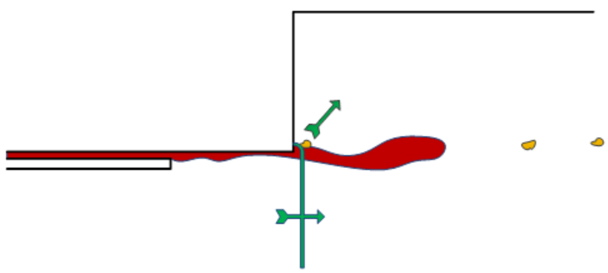
A detailed computational investigation by Harvazinski et al. of the single element configuration has provided useful insight into the longitudinal instability mechanism by comparing simulations for stable and unstable conditions [4]. For the stable length the heat release from combustion was continuous throughout the cycle. The two unstable cases showed different instability mechanisms. Figure 2 shows a sketch of the key aspects of the instability mechanism. For both of the cases the fuel supply into the combustor is temporarily disrupted (2a). This disruption is the result of the longitudinal pressure pulse traveling upstream into the oxidizer post displacing the incoming fuel. There is also an increase in mixing in the cup region from increased vorticity generated from baroclinic torque present because of the moving pressure gradient. This allows lingering fuel in the cup region to burn before entering the combustor, further enhancing the fuel supply cut off.

Following the fuel cut off there is a concomitant reduction in the burning in the combustor. As the fuel recovers from the cut off, it begins to reenter the combustor but does so without burning because the combustion has been interrupted at the head end. This results in an

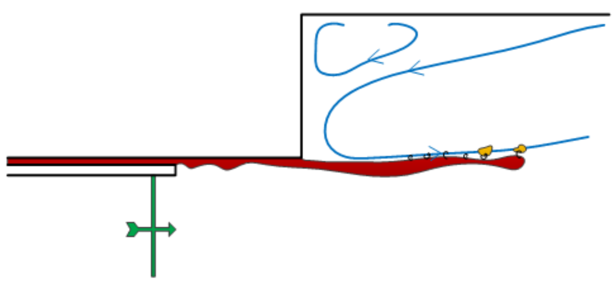
accumulation of unburnt fuel in the combustor head end. The re-ignition of the accumulated fuel occurs in one of two different ways, and the selection of the reignition mechanism is dependent on the oxidizer post length. The post-coupled mechanism occurs for the intermediate length case. Here, the returning pressure pulse in the oxidizer post pushes the accumulated fuel into the warm recirculated cases and results in a massive ignition event and rapid burning of the accumulated fuel. In the vortex coupled mechanism, which occurs for the longest length, the timing of the pressure pulse is not a key factor. Instead, after spending sufficient time mixing with the warm recirculating gases, re-ignition takes place spontaneously downstream from the dump plane instead of the close to the dump plane [4].



(a) Fuel cut off event.



(b) Fuel re-ignition event, post coupled mechanism.



(c) Fuel re-ignition event, vortex transport mechanism.

Figure 2: Sketch of the fuel cut off and both re-ignition behaviors. In all drawings the red region represents accumulated fuel, the gold regions are heat release, and the green line is the longitudinal pressure pulse in the oxidizer tube.

MULTI-ELEMENT STUDIES

Single element experiments are attractive for simulations because the physical size and geometric complexity is typically such that the problem that can be run on current generation computers. Single element studies do have some limitations in their applicability to actual engines. First the inter-element interaction is missing; any coupling between injectors cannot be captured in the single element case. Second the effect of the walls is exaggerated. For these reasons it is important to also evaluate CFD tools on multi-element configurations. The configuration selected for this work is the transverse instability combustor (TIC) from Purdue University. The experiment is designed to study the response of injectors to high-amplitude velocity fluctuations which are present in transverse combustion instabilities [11]. A linear arrangement of seven elements in a rectangular chamber is used; the center element is referred to as the study element and is subjected to the highest transverse velocity fluctuations. To date, data has been collected using four versions of the TIC; the configurations are overviewed in the following section. Simulations have focused on two of these configurations and it is planned to investigate the latest iteration in the future.

EXPERIMENTAL STUDIES

To date four iterations of the TIC have been used to study transverse instabilities. The initial configuration was originally designed by Pomeroy. Table I shows the relevant details of each of the configurations. All experiments use decomposed hydrogen peroxide as the oxidizer while the fuel type varies. The first two configurations use different fuels for the driving elements and the study element while in the final two configurations the same fuel is used for both. The oxidizer injectors are all choked. Different type of choking including, perforated plates, choked slots, and choked venturis are used. The geometry for TIC 1a and 1b is shown in Figure 3. Geometrically the main difference between TIC 1a and 1b is the study element. In TIC 1b the study element uses the same choked slots configuration that was used in the prior single element studies. All of the cases have a rectangular chamber with a nozzle attached to the downstream end. High frequency pressure transducers are used to record the pressure at various locations in the chamber [11, 12, 13, 14, 15, 16, 17].

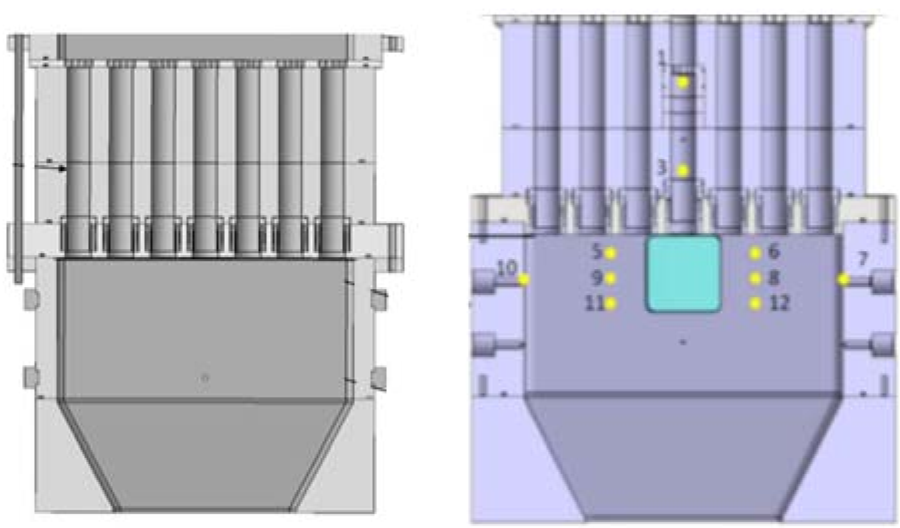
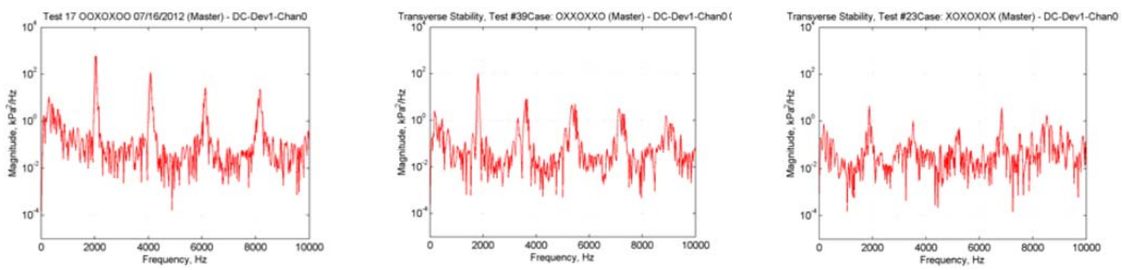


Figure 3: TIC revision 1a (left) and revision 1b (right).

Table I: Overview of the four TIC experiments.

		TIC 1a	TIC 1b	TIC 1c	TIC 1d
Oxidizer		H_2O_2	H_2O_2	H_2O_2	H_2O_2
Fuel	Driving	JP-8	RP-1	CH_4	CH_4
	Study	$C_{12}H_{26}$	C_2H_6	CH_4	CH_4
Oxidizer Inlet	Driving	Perforated Plate	Perforated Plate	Perforated Plate	Choked Venturi
	Study	Perforated Plate	Choked Slots	Choked Slots	Choked Venturi
Notes		Two-phase flow		Multiple study ox-post lengths considered	Multiple ox-post lengths considered
Companion Simulations			3-element	7-element	Future Work

The amplitude of the transverse instability can be controlled in two ways. The first way is exercised in TIC 1a and 1b does so by selecting which elements flow fuel and oxidizer and which elements flow only oxidizer. The second way used in TIC 1c and 1d seeks to control the amplitude by varying the oxidizer post lengths as in the single-element studies. The result is the ability to vary the amplitude between 5% and 70% of the chamber pressure (nominally 800 kPa in TIC 1b), although the former method seems to work more repeatability than the latter method. In TIC 1a it was found that the greatest amplitudes resulted from having all elements flow both fuel and oxidizer. When only the outer and study elements flowed fuel and oxidizer the amplitude was reduced by a factor of 2. The lowest amplitude resulted from an asymmetric configuration where the left three elements had both fuel and oxidizer and the right four elements had oxidizer only. PSD amplitude results from three different TIC 1b tests are shown in Figure 4.



(a) (b) (c)

Figure 4: PSD plots from TIC 1b.

The amplitude of TIC 1c was controlled by adjusting the driving oxidizer post lengths while holding the length of the study element fixed. It was found that the length of the driving oxidizer post length did not have a strong effect on the amplitude. The 4.4 inch length which corresponds to the unstable single element length was found to be unstable while the other lengths tested were all stable. Moreover, the results for the 4.4 inch case were not always repeatable with stable amplitudes obtained in some of the tests. It is hypothesized that the unstable study element length may be responsible for dampening the overall instability amplitude of the chamber. Amplitudes for TIC 1c are shown in Figure 5.

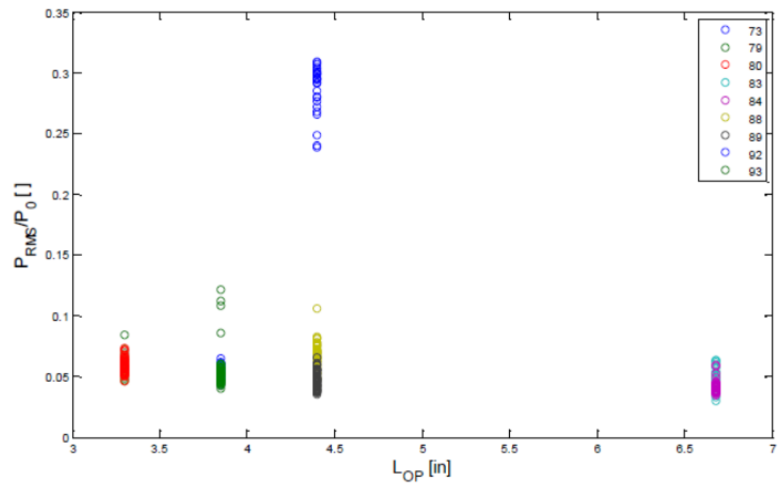


Figure 5: amplitudes from TIC 1c [17].

COMPUTATIONAL STUDIES

There are two distinct computational approaches that can be taken to study the transverse instability. Both approaches have been modeled by Shipley et al for different conditions. Example computational domains for each scenario are shown in Figure 6. The first approach is to replace the driving injectors with a forced-wall boundary condition which can control the level of transverse instability without relying on the physics of the driving injectors [18]. This allows the amplitude of the transverse instability to be controlled. In the case where artificial forcing is used, the simulation is useful to determine how the study element responds to transverse oscillations. This method requires substantially less computational resources because not all of the injectors need to be modeled, but it has the limitation that inter-element interactions are not captured. It is possible that the overall stability characteristics of the study element are dependent on the interaction between other elements and this would not be captured. In the second approach, all seven injector elements are simulated [19]. This approach can be very expensive because of the mesh size requirements, but the computation is now able to predict inter-element interactions as well as the self-excited nature of the instability. In other words, the amplitude of the instability is not pre-specified and is the result of the operating conditions. On the other hand, this approach does make it difficult to analyze or assess the response of the study element because the physics of the center element is tightly coupled with the physics of the entire combustor.

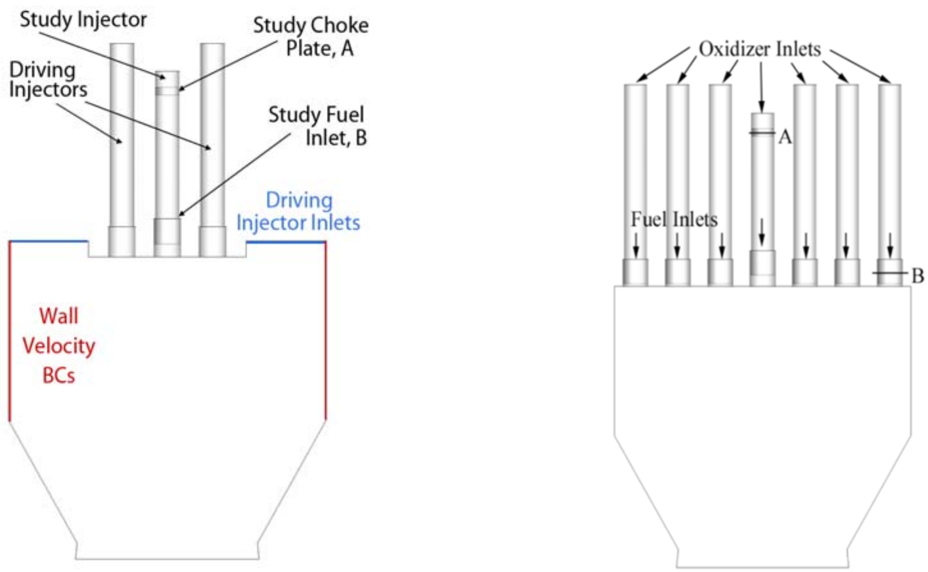


Figure 6: Computational domains for the artificially forced case (left) and the self-excited case (right).

COMPUTATIONAL STUDIES – ARTIFICIAL FORCING

The TIC 1b chamber was modeled by Shipley et al. using the artificial forcing approach. The computational domain was previously shown in Figure 6. Only three of the seven elements are included. The two outside elements on each side responsible for the driving are replaced with constant mass flow boundary conditions. The constant mass inflow conditions are based on equilibrium temperature and combustion products. The elements on either side of the study element flow only flow oxidizer. A key aspect of this work is that the study element is very similar to the one used in the single-element longitudinal studies. The study element flows decomposed hydrogen peroxide oxidizer though a series of choked slots. The fuel, ethane, is injected through a shear coaxial injector. To model the acoustic forcing in the chamber a velocity perturbation is introduced on the vertical side walls of the chamber. The perturbation follows the form,

$$u_{\mathrm{wall}} = A \sin(2\pi f + \varphi) \quad (1)$$

where A is the prescribed amplitude, f is the prescribed frequency and φ is introduced to allow for a potential phase difference between the two walls. This approach has also been used to replicate the disturbance from an acoustic source [20]. For the present study to generate the 1W wave, the two walls are set to fluctuate in phase ($\varphi = 0°$). By adjusting the prescribed amplitude, it is possible to generate different 1W amplitudes in the chamber. A three-point parametric study, shown in Figure 7, demonstrates how the 1W amplitude varies with respect to the prescribed amplitude. A limitation of this simple disturbance function is that only the amplitude of the first mode can be tuned, and any higher-order harmonics will be the result of physical excitation. An alternate forcing function would be to prescribe a series of sine waves for the higher-order harmonic frequencies, which would allow for a better match of the experimental operating conditions but this has not yet been used in the computational studies.

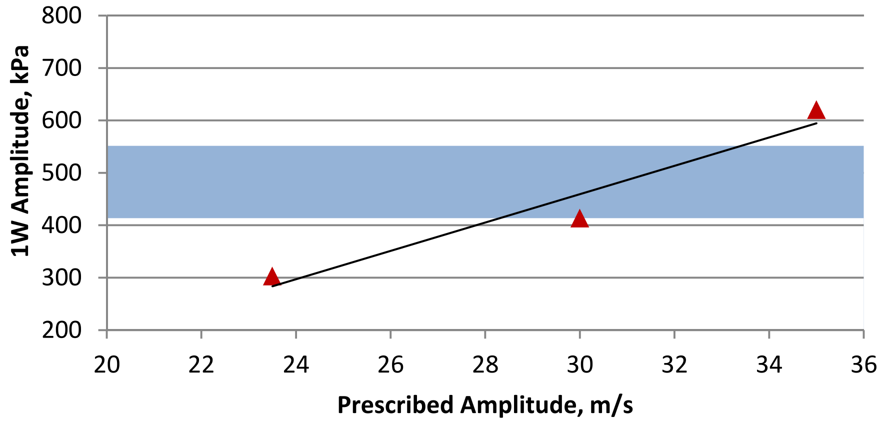


Figure 7: Effect of wall velocity on pressure. The blue band represents the experimental pressure fluctuations filtered at the primary acoustic mode [21].

A wall-forcing amplitude of 30 m/s was found to be give reasonable agreement with the experimental dataset [18]. A comparison of the PSD for the computational data and experimental data is shown in Figure 8. Amplitudes and frequencies for the first four modes are in excellent agreement with the experimental data. Because the chamber is forced at a prescribed frequency and amplitude a match in the first mode frequency and amplitude is guaranteed given appropriate tuning. It is interesting that for this case despite the fact that only a single amplitude and frequency was prescribed, the artificial forcing is able to replicate reasonably well the frequency and amplitudes of the higher-order harmonics.

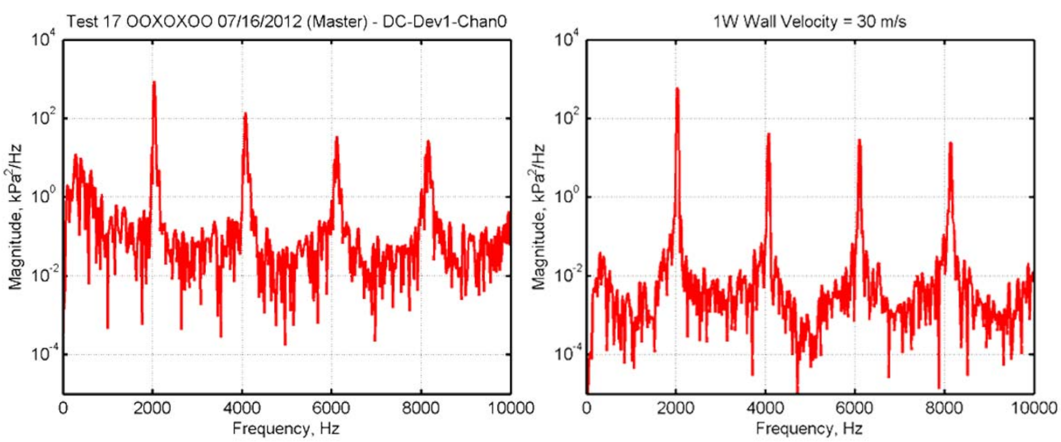


Figure 8: Power spectral density plots for the experiment (left) and simulation (right) [21].

A series of unsteady images of the flowfield is shown in Figure 9. The figure shows the pressure, temperature and heat release over the course of a single cycle. The complete cycle is defined as the pressure pulse moving from the left side to the right side and then back. At the first time instance, the pressure pulse is moving from left to right. As the pressure pulse travels past the injectors a secondary compression wave moves upstream in the injectors. The center study element lies at a pressure node and as a result there is considerably less coupling between the pressure waves in the oxidizer post and the main chamber pressure waves compared to the elements on either side of the study element. This behavior is evident in the time series plot. Recall that in the single element experiment the wave in the oxidizer post is a key feature of the instability mechanism.

Across the transverse wave there is also an increase in the temperature. When the wave reaches the right wall (time-slice 2), it reflects back to the left due to the wall boundary condition. The wave then passes through the center of the chamber, which disrupts the heat release. Once the wave hits the left wall it is reflected back to the right and the cycle repeats. The heat release disruption can clearly be seen in slices 4 and 5 with the wave moving from left to right. Once the wave finishes passing through the center there is an increase in heat release before the wave is reflected back.

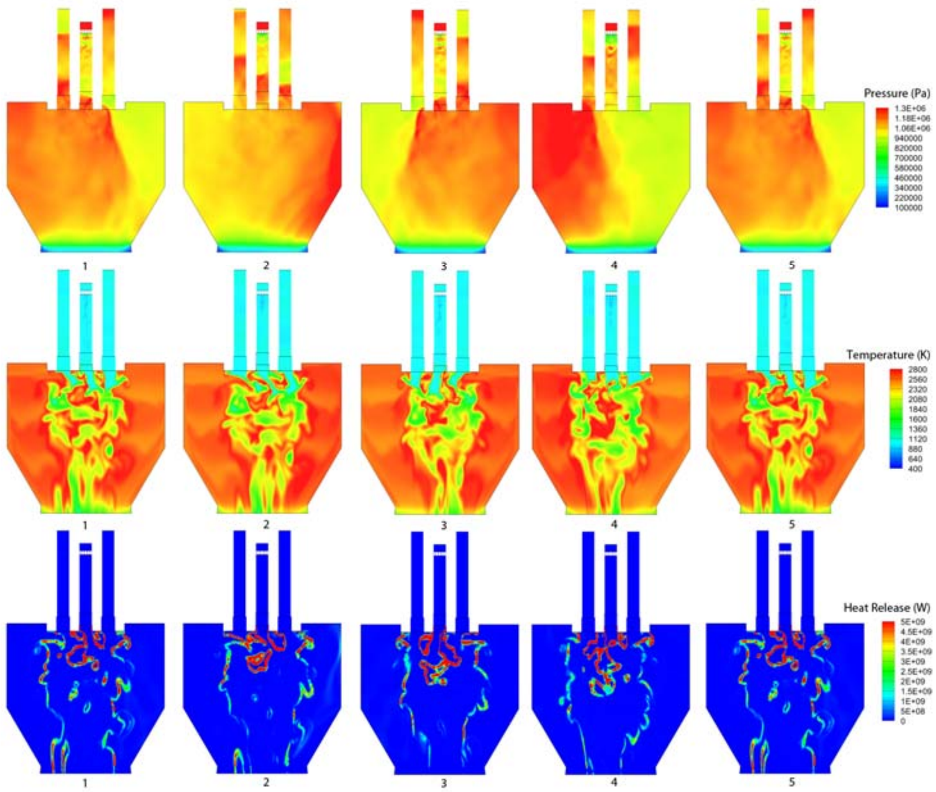


Figure 9: Cycle analysis of the unsteady flowfield. Columns represent the flowfield at the same times [18].

COMPUTATIONAL STUDIES – SELF-EXCITED FORCING

Shipley et al. also simulated all seven elements of the TIC 1c for a slightly different operating condition. Methane is used as the fuel in both the study and driving elements (Shipley 2014). The study element again features the choked slots and is similar to the element under investigation in the single element studies. The outside elements used a choke plate to acoustically isolate the chamber from the oxidizer manifold. This boundary condition was

approximated with a constant mass flow boundary. The approximate boundary condition has been shown previously in single element work to reasonable replicate the effects of the choked inlet [ref]. The computational domain is shown in Figure 6 and a representative unsteady flowfield is shown in Figure 10. The flowfield which shows the fuel and oxygen mass fractions shows substantial differences in the area around each of the injectors. The outer injectors show regions of increased fuel concentration which extend further downstream compared to the center element. The same is true for the oxidizer core. Note that the elements on other side of the study element do not flow fuel, this likely indicates why less fuel is seen on either side of these elements since there is excess oxidizer available to mix with and ultimately burn.

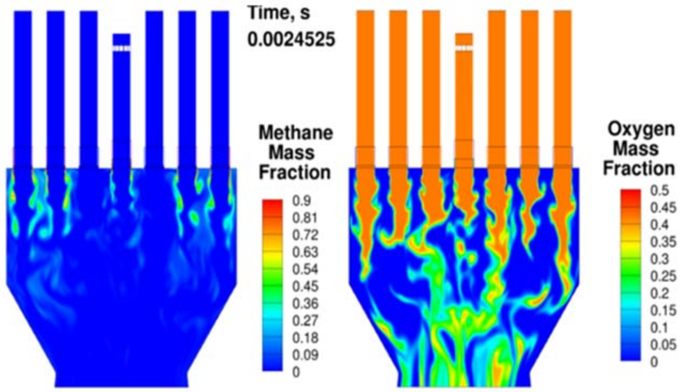


Figure 10: Flowfield [19].

The full simulation shows a significant transient behavior and never reaches a limit cycle like the experiment, the transient behavior of the pressure as measured from the sidewall is shown in Figure 11. It is possible that the simulations need to be run for a longer period of time. The temporal evolution of the simulation can be divided into several distinct regimes. Ignition takes place in the initial startup segment, following this there is a low amplitude instability period where peak-to-peak fluctuations are on the order of 10-20% of the mean pressure (970 kPa). The period of low instability is followed by a ramp up in amplitude with peak-to-peak amplitudes reaching 70% of the mean pressure. After reaching the maximum amplitude the instability decays. The period after the decay is mixed with high and low amplitude cycles.

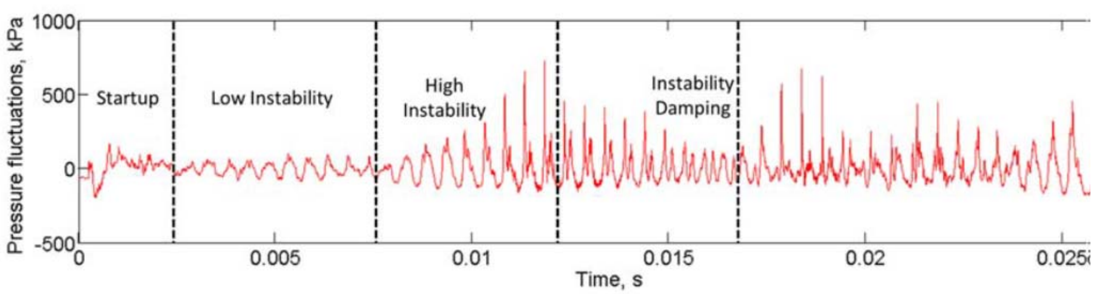


Figure 11 transient behavior, measurement location is on the left sidewall [21].

In addition to changes in the amplitude the frequency content is also different in each regime. During the low amplitude time the 1W mode is excited and higher-order modes have lower amplitudes. As the instability amplitude increases the higher-order harmonics become excited, the wave forms then display the classic wave-steepening phenomena associated with high amplitude instabilities. The increased amplitude of the higher-order modes remains in the instability damping phase as well. Figure 12 shows the pressure pulse movement in the full chamber. This wave is shown during the high instability phase. There are secondary longitudinal waves present in each of the side elements, which is an effect that cannot be captured by the artificially forced model and is likely significant for predicting the amplitude of the instability. The

injectors on the sides act as half-wave resonators which couple with the 1T mode in the chamber. The beginning (8.8275 ms) and the mid-point (9.0900 ms) show mirror images. When the pressure in the chamber is high on the right side the pressure at the downstream end of the right injectors is also high. Like the reduced model there is less coupling between the center element and the chamber compared to the outside elements.

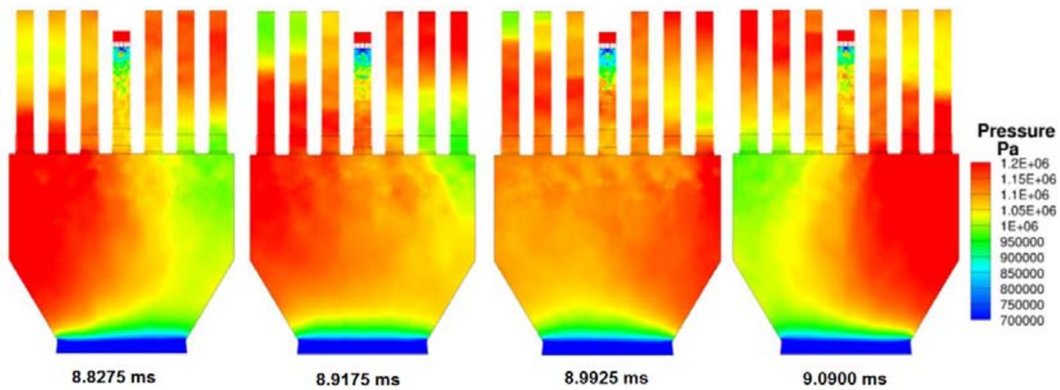


Figure 12 pressure wave during the high instability phase. Notice the secondary waves present in the side elements.

The simulation of the full seven element combustor reveals important insights into the instability mechanism. Vortex impingement is prevalent during the high-amplitude instability portion of the simulation. The impingement can be divided into two types---impingement with the wall that occurs with the outer injectors, and impingement with other jets. Figure 13 shows the vortex impingement that takes place during the high amplitude instability phase. Fuel entrained vortices are shed from the injector lip (detail 1b). In the outside injector as the transverse pressure wave passes by each injector the vortex impinges on the outside wall. This can be seen in detail 2b. Notice that the heat release is significantly amplified when the pressure is higher; this can be seen by comparing details 1c and 2c. There also is vortex impingement between the elements; this is highlighted in detail 2b.

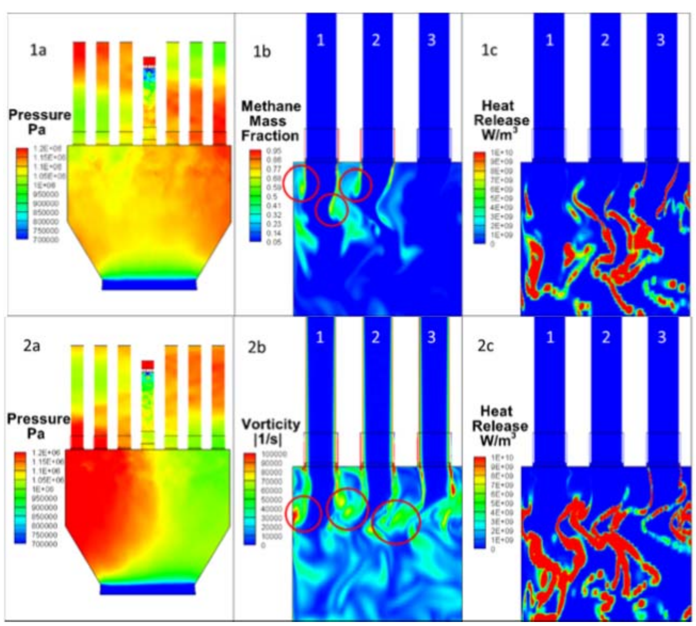


Figure 13: Vortex interaction between elements and between the side elements and the outside wall.

SUMMARY AND CONCLUSIONS

An overview of computational approaches for modeling an experimental sub-scale transverse combustor was presented. The combustor can generate instability amplitudes that range from 5% to 70% of the mean chamber pressure. Two different modeling approaches are used to study transverse instabilities. The first approach is a simplified model which replaces the driving injectors with an artificial forcing source. Using this approach one can study the impact of transverse instability on the central study element. Because the simulated forcing, one is able to precisely control the level and frequency of the instability. The simplified model does not account for the self-excited nature of the experiment and inter-element interactions are not fully captured. To address both of these missing features a second simulation approach was under-taken where all seven-elements were modeled. This second approach captures self-excited instabilities and shows that vortex impingement is amplified by the transverse waves. Both approaches provide useful insights into aspects of transverse mode instability in rocket combustors.

REFERENCES

[1] J. Oefelein and V. Yang, "Comprehensice Review of Liquid-Propellant Combustion Instabilities in F-1 Engines," *Journal of Propulsion and Power,* pp. 657-677, 1993.

[2] R. Garby, L. Selle and T. Poinsot, "Large-Eddy Simulation of Combustion Instabilities in a Variable-length Combustor," *Comptes Rendus Mécanique,* vol. 341, no. 1-2, pp. 220-229, 2013.

[3] M. Harvazinski, W. Anderson and C. Merkle, "Analysis of Self-Excited Combustion Instability using Two- and Three-Dimensional Simulations," *Journal of Propulsion and Power,* vol. 29, no. 2, pp. 396-409, 2013.

[4] M. Harvazinski, C. Huang, V. Sankaran, T. Feldman, W. Anderson, C. Merkle and D. Talley, "Coupling between hydrodynamics, acoustics, and heat release in a self-excited unstable combustor," *Physics of Fluids,* vol. 27, p. 045102, 2015.

[5] S. Sardeshmukh, W. Anderson, M. Harvazinski and V. Sankaran, "Prediction of Combustion Instability with Detailed Chemical Kinetics," in *AIAA SciTech 2015*, Orlando, FL, AIAA Paper No. 2015-1826, 2015.

[6] R. Smith, G. Xia, W. Anderson and C. Merkle, "Computational studies of the effects of oxidizer injector length on combustion instability," *Combustion Theory and Modeling,* vol. 12, pp. 341-368, 2012.

[7] R. Smith, G. Xia, W. Anderson and C. L. Merkle, "Computational Simulations of the Effect of Back step Height on Non-premixed Combustion Instability," *AIAA Journal,* vol. 48, no. 9, pp. 1857-1868, 2010.

[8] S. Srinivasan, R. Ranjan and S. Menon, "Flame Dynamics Durning Combustion Instability in a High-Pressure, Shear-Coaxial Injector Combustor," *Flow, Turbulence and Combustion,* vol. 94, no. 1, pp. 237-262, 2015.

[9] K. Miller, J. Sisco, N. Nugent and W. Anderson, "Combustion Instability with a Single-Element Swirl Injector," *Journal of Propulsion and Power,* vol. 23, no. 5, pp. 1102-1112, 2007.

[10] Y. Yu, J. Sisco, S. Rosen, A. Madhav and W. Anderson, "Spontaneous Longitudinal Combustion Instability in a Continuously-Variable Resonance Combustor," *Journal of Propulsion and Power,* vol. 28, no. 5, pp. 876-887, 2012.

[11] B. Pomeroy, W. Lamont and W. Anderson, "Subscale Tool for Determining Transverse Combustion Response," in *AIAA JPC Conference*, Denver, CO, AIAA Paper No. 2009-5490, 2009.

[12] B. Pomeroy, J. Sisco, J. Eckstein and W. Anderson, "Characterization of Gas-Centered Swirl-Coaxial Injector Stability in a Subscale Multi-Element Combustor," in *AIAA JPC Conference*, Hartford, CT, AIAA Paper No. 2008-5249, 2008.

[13] B. Pomeroy, N. Nugent and W. Anderson, "Measuring Transverse Stability Combustion

Response at Full Scale Frequencies in a Subscale Combustor," in *AIAA JPC Conference*, Nashville, TN, AIAA Paper No. 2010-7146, 2010.

[14] B. Pomeroy, C. Morgan and W. Anderson, "Response of a Gas-Centered Swirl Coaxial Injector to Transverse Instabilities," in *AIAA JPC Conference*, San Diego, CA, AIAA Paper No. 2011-5698, 2011.

[15] M. Wierman, N. Nugent and W. Anderson, "Combustion response of a LOX/LCH4 element to transverse instabilities," in *AIAA JPC Conference*, San Diego, CA, AIAA Paper No. 2011-5549, 2011.

[16] M. Wierman, W. Hallum and W. Anderson, "Stability Characterization of a High Pressure Transverse Combustor with Discretely Variable Oxidizer Post Lengths," in *AIAA JPC Conference*, Cleveland, OH, AIAA Paper No. 2014-3774, 2014.

[17] M. Wierman, "Measurement of Combustion Response to Transverse Modes at High Pressure," Purdue University, West Lafayette, 2015.

[18] K. Shipley, C. Morgan, W. Anderson, M. Harvazinski and V. Sankran, "Computational and Experimental Investigation of Transverse Combustion Instabilities," in *AIAA Joint Propulsion Conference*, San Jose, CA, AIAA Paper No. 2013-3992, 2013.

[19] K. Shipley, W. Anderson, M. Harvazinski and V. Sankaran, "A Computational Study of Transverse Combustion Instability Mechanisms," in *AIAA Joint Propulsion Conference*, Cleveland, OH, AIAA Paper No. 2014-3680, 2014.

[20] T. Schmitt, J. Rodriguez, I. Leyva and S. Candel, "Experiments and Numerical Simulations of Mixing Under Supercritical Conditions," *Physics of Fluids,* vol. 24, 2012.

[21] K. Shipley, "Multi-injector Modeling of Transverse Combustion Instability Experiments," Purdue University, West Lafayette, 2013.